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The influence of material properties, the geometric shape of the section and temperature on the stressed state of the working bodies

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ABSTRACT

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In the work on stretching a matrix with a rigid inclusion -an ideal-plastic cylindrical waveguide, a technique was developed for determining the thickness of the boundary layer, which can be generalized and extended to all problems.

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1. Introduction

In this paper, we consider the influence of edge effects arising when pipes and pipelines of a finite length are immersed in another medium. It is assumed that one end or both ends of a pipe or pipeline (hereinafter inclusions) having a finite length is immersed in some foreign medium (hereinafter matrix) in the well and boundary value problems are considered about the current stress state of pipes, pipelines and rods [1-3] ...

In the following, we continue to study the boundary value problems considered in the previous work, taking into account the following additions [4-5]:

- changes in the shape of the cross-section of the pipe - inclusions;
- changes in the properties of the inclusion material and matrix - rock;
- the influence of the thermal effects of the matrix.

2. Research methods

Consider the singular problem of stretching an elastic plane along a thin rigid inclusion of high thermal conductivity and finite width $2l$, coupled to the rest of the body by the matrix at

all points of the lateral surface, i.e. conjugate to the matrix. In this case, when the Young's modulus of the inclusion $2l$ is much greater than its thickness δ , i.e. $\lambda = E^+ / E^- \gg 1, \delta / e \ll 1$, where E^+ and E^- - are Young's module of the inclusion and the matrix, respectively, near the inclusion in the matrix there is a boundary layer in which only the shear stress τ_{12} is significant, and all other voltages are negligible [6].

Let us assume that the inclusion is located along the segment $x_2 = 0, |x_1| < l$ in the matrix stretched at infinity by the stress $\tau_{11} = P^{11}$ along the inclusion (Fig. 1).

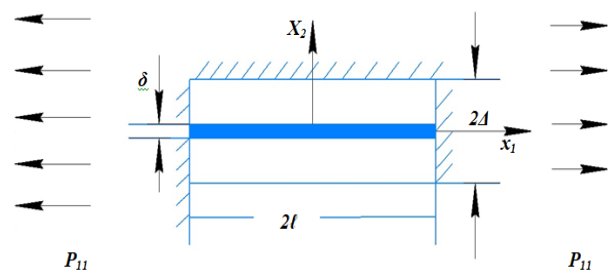


Fig. 1.

The general solution of the boundary layer equations in the matrix is as follows:

$$u_1 = V + \frac{\tau_{12}}{\mu^-} x_2, \quad \tau_{12} = \tau = \text{const} \quad (1)$$

Here ν and τ are arbitrary functions x_1 , μ is the matrix shift modulus.

As you can see, the stress τ_{12} is constant in the cross-section of the boundary layer and is equal to the shear stress $\tau(x_1)$ on the banks of the inclusion, and the displacement u_1 is linear (function) x_2 .

At $x_2=0$, the boundary layer is associated with an inclusion. Due to rigid adhesion to the matrix, the displacement u_1 of the matrix at this point should be the same as the displacement u_1 of the inclusion.

Therefore, $V=V(x_1)$ in formulas (1) is the displacement of the inclusion at the corresponding points.

At the boundary of the boundary layer at $x_2=\pm A$, the field in the boundary layer (1) must be "stitched" with the external, unperturbed field $u_1 = x_1 \rho_{11} E^-$ as a condition for "stitching" it is natural to require the equality of the longitudinal deformation $\varepsilon_{11} = \partial u_1 / \partial x_1$ and another field at the boundary of the boundary layer at $x_2 = \pm \Delta$. As a result, based on (1), we obtain

$$V'(x_1) + \frac{\Delta}{\mu^-} \tau'(x_1) = \frac{P_{11}}{E^-} \quad (2)$$

Together with the equations

$$\tau = E^+ V(x_1), \quad \tau = -1/2 \partial \dot{t}(x_1) \quad (3)$$

we obtain a closed system of ordinary differential equations for the functions V , τ and τ .

The solution to this system gives the desired solution to the problem posed in the boundary layer approximation [7].

The structure of the field is shown in Fig. 1: the inside of the rectangle $|x_1| \leq l, |x_2| \leq \Delta$ is the boundary layer, the outside of the same rectangle is the unperturbed field.

First, we exclude $V'(x_1)$ from system (2), (3), we find

$$\tau = E^+ \left(\frac{P_{11}}{E^-} - \frac{\Delta}{\mu^-} \tau'(x_1) \right) \quad (4)$$

Now we substitute τ from the second equation in (4) into the first, we obtain

$$\frac{\tau(x_1)}{E^+} = \frac{P_{11}}{E^-} + \frac{\delta \Delta}{2\mu^-} \tau''(x_1) \quad (5)$$

Here, the thickness δ is assumed to be independent of x_1

The general solution of this second-order differential equation has the form:

$$\tau = C_1 l^{\gamma x_1} + C_2 l^{-\gamma x_1} + \frac{P_{11}}{\Delta \delta \gamma^2 (1 + \nu^-)} \quad (6)$$

where

$$\gamma = \sqrt{\frac{2\mu^-}{\Delta \delta E^-}} \quad (7)$$

Here C_1 and C_2 are arbitrary constants.

3. Results and discussion

The inclusion can be subjected to direct external loads applied to it. Consider the following four options loaded (Fig. 2).

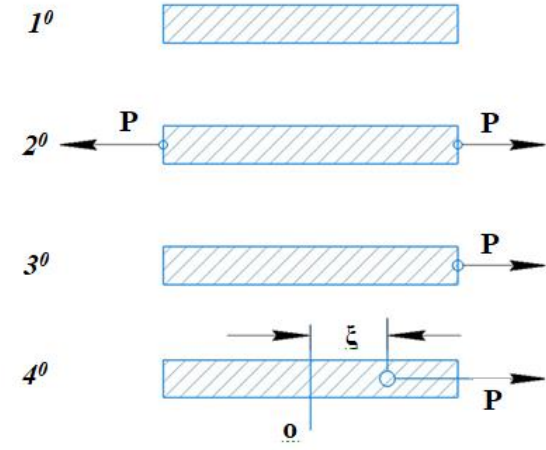


Fig. 2.

1. Switching on is free from external loads. In this case, the switch-off ends can be considered unfinished, so that the voltage τ at the ends is equal to zero

$$\text{at } x_1 = \pm l \quad \tau = 0 \quad (8)$$

2. External forces are applied to both ends of the inclusion. In this case, we have

$$\text{at } x_1 = \pm l \quad \tau = \frac{P}{\delta} \quad (9)$$

where p is the force per unit of thickness (normal to Fig. 2).

3. Force P is applied to only one end.

In this case, the following boundary conditions will be

$$\begin{aligned} \text{at } x_1 = -l \quad \tau &= 0 \\ \text{at } x_1 = \pm l \quad \tau &= \frac{P}{\delta} \end{aligned} \quad (10)$$

4. The force P is applied at some point $x_1 = \xi$ of the inclusion. In this case, the solution has a discontinuity at the point $x_1 = \xi$; find

$$\begin{aligned} \text{at } x_1 = \pm l \quad \tau &= 0 \\ \text{at } x_1 = \xi \pm 0 \\ (\tau(\xi + 0) - \tau(\xi - c)) &= \frac{P}{\delta} \end{aligned} \quad (11)$$

The linear superposition of these four cases allows us to study the general case of an arbitrary external load on the inclusion [8].

We present the values of the constants C_1 and C_2 in the general solution (6) for the first three cases, obtained as a result of simple calculations:

1. Switching on is free from external loads, only the external p_{11} field in the matrix acts

$$C_1 = C_2 = \frac{-P_{11}}{2\Delta\delta\gamma^2(1+\nu^-)ch\gamma e} \quad (12)$$

2. Tensile forces P are applied to both ends of the inclusion, there is no external field in the matrix ($p_{11}=0$):

$$C_1 = C_2 = \frac{P}{2\delta ch\gamma e} \quad (13)$$

3. Force P is applied only to the right end of the switch, there is no external field in the matrix ($p_{11}=0$):

$$C_1 = \frac{P}{4\delta} \left(\frac{C_1}{ch\gamma l} + \frac{J_1}{jh\gamma l} \right)$$

$$C_2 = \frac{P}{4\delta} \left(\frac{1}{ch\gamma l} + \frac{1}{jh\gamma l} \right) \quad (14)$$

In the case of the action of the internal force P at the point $x_1 = \xi$, the solution consists of two analytical parts (piecewise analytical solution):

$$\tau = \begin{cases} V_1 + \frac{1}{\gamma E^+} (C_1 e^{\gamma x_1} - C_2 e^{-\gamma x_1}) \cdot p \quad -l < x_1 < \xi \\ V_2 + \frac{1}{\gamma E^+} (D_1 e^{\gamma x_1} - D_2 e^{-\gamma x_1}) \cdot p \quad \xi < x_1 < l \end{cases} \quad (15)$$

Here V_1 and V_2 are some constants; only one of them can be set equal to zero, since it is determined by the displacement of the body as a whole. Therefore, condition (15) does not provide an additional condition for determining the voltage field in the inclusion.

Note that the equilibrium condition for the entire inclusion as a whole must always be satisfied [9]

$$2 \int_{-l}^{+l} \tau(x_1) dx_1 = P \quad (16)$$

It's not hard to check. that in all previously considered cases 1, 2, 3, the equilibrium condition as a whole is satisfied automatically.

Using solution (14) and (15), we present the following convenient formulas:

$$\begin{cases} C_1 = \frac{1}{4} \tau(\xi-0) \left(\frac{1}{jh\gamma(l+\xi)} + \frac{1}{ch\gamma(l+\xi)} \right) \\ C_2 = \frac{1}{4} \tau(\xi-0) \left(\frac{1}{ch\gamma(l+\xi)} - \frac{1}{\xi h\gamma(l+\xi)} \right) \end{cases} \quad (17)$$

$$\begin{cases} D_1 = \frac{1}{4} \tau(\xi-0) \left(\frac{1}{ch\gamma(l-\xi)} - \frac{1}{\xi h\gamma(l-\xi)} \right) \\ D_2 = \frac{1}{4} \tau(\xi-0) \left(\frac{1}{ch\gamma(l-\xi)} + \frac{1}{\xi h\gamma(l-\xi)} \right) \end{cases} \quad (18)$$

In the considered case of the action of an internal force, on the basis of (2), (3) and (18), one should additionally assume that the function $\tau(x_1)$ is continuous at the point $x_1 = \xi$ of application of an external force P , i.e.

$$\tau(\xi-0) = \tau(\xi+0) \quad (19)$$

Using the second relation (3) and (15), from this we find

$$\tau'(\xi-0) = \tau'(\xi+0) \quad (20)$$

and

$$C_1' l^{\gamma \xi} - C_2' l^{-\gamma \xi} = D_1 l^{\gamma \xi} - D_2 l^{-\gamma \xi}$$

On the other hand, conditions (11) based on (15) give

$$(D_1 l^{\gamma \xi} + D_2 l^{-\gamma \xi}) - (C_1' l^{\gamma \xi} + C_2' l^{-\gamma \xi}) = \frac{P}{\delta}$$

$$C_1' l^{\gamma \xi} + C_2' l^{-\gamma \xi} = 0$$

$$D_1 l^{\gamma \xi} + D_2 l^{-\gamma \xi} = 0 \quad (21)$$

The solution of system (20) and (21), consisting of four linear equations, can be written in the following final form:

4. Force P is applied at the internal point $x_1 = \xi$ of the inclusion, there is no external field in the matrix ($p_{11} = 0$)

$$C_1^{(\xi)?) = \frac{P}{2\delta} \left(e^{\gamma \xi} \frac{e^{2\gamma l} - e^{-2\gamma \xi}}{1 - e^{4\gamma l}} - e^{-\gamma \xi} \right) \quad (22)$$

$$D_1^{(\xi)?) = \frac{P}{2\delta} \cdot e^{\gamma \xi} \cdot \frac{e^{2\gamma l} - e^{-2\gamma \xi}}{1 - e^{4\gamma l}}$$

$$C_1^{(\xi)?) = \frac{P}{2\delta} \cdot e^{-2\gamma \xi} \left(e^{-\gamma \xi} - e^{\gamma \xi} \frac{e^{2\gamma l} - e^{-2\gamma \xi}}{1 - e^{4\gamma l}} \right)$$

$$D_1^{(\xi)?) = -\frac{P}{2\delta} \cdot e^{\gamma(2l+\xi)} \cdot \frac{e^{2\gamma l} - e^{-2\gamma \xi}}{1 - e^{4\gamma l}}$$

$$\tau = C_1' e^{\gamma x_1} + C_2' e^{-\gamma x_1} \quad \text{at } -l < x_1 < \xi$$

$$\tau = D_1 e^{\gamma x_1} + D_2 e^{-\gamma x_1} \quad \text{at } \xi < x_1 < l$$

The general qualitative picture of the behavior of the functions $V(x_1)$, $\tau(x_1)$, and $\tau(x_1)$ in this case, according to (22), is shown in Fig. 3. the functions $V(x_1)$ and $\tau(x_1)$ are continuous everywhere, but have a break at the point $x_1 = \xi$ (jump of the derivative). The function $\tau(x_1)$ has a discontinuity of the first kind at the point $x_1 = \xi$.

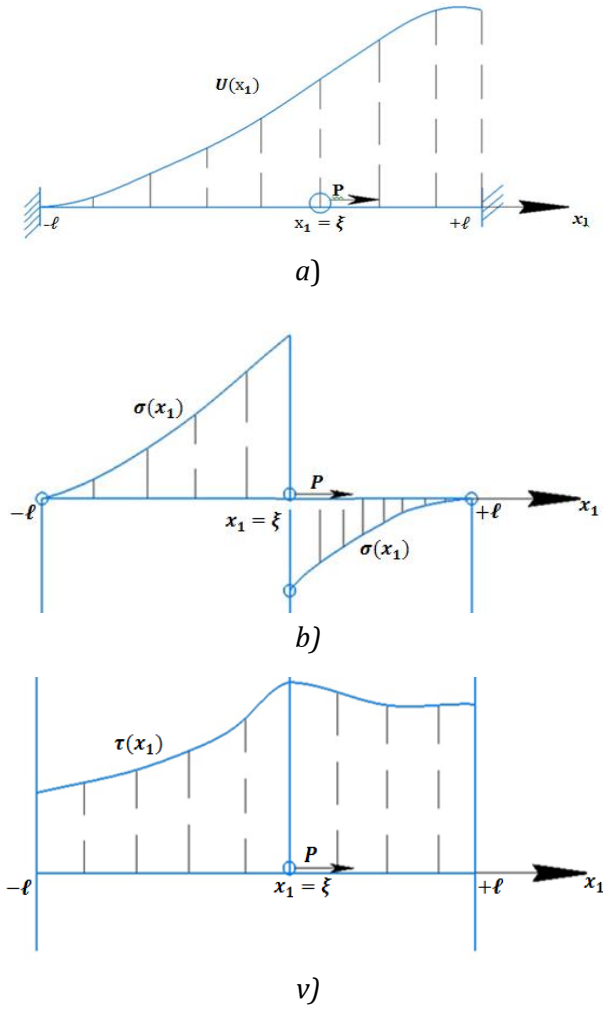


Fig. 3

Solution (22) allows solving the problem of an arbitrarily distributed external load in the inclusion $\rho = \rho(x_1)$. For this, it is necessary to replace p by $p d \xi$ in formulas (22) and integrate over ξ from $-l$ to l . We get:

$$\tau = \frac{1}{2\delta} \int_{-1}^{+1} p(\xi) \left(e^{\gamma x_1} \left(e^{\gamma \xi} \frac{e^{2\gamma l} - e^{-2\gamma \xi}}{1 - e^{4\gamma l}} - e^{-\gamma \xi} \right) + e^{-\gamma x_1} \cdot e^{-2\gamma \xi} \left(e^{-\gamma \xi} - e^{\gamma \xi} \frac{e^{2\gamma l} - e^{-2\gamma \xi}}{1 - e^{4\gamma l}} \right) \right) d\xi$$

$(-l < x_1 < \xi)$ (23)

$$\tau = \frac{1}{2\delta} \int_{-1}^{+1} p(\xi) \left(e^{\gamma(x_1 + \xi)} \cdot \frac{e^{2\gamma l} - e^{-2\gamma \xi}}{1 - e^{4\gamma l}} - e^{\gamma(2l - x_1 + \xi)} \cdot \frac{e^{2\gamma l} - e^{-2\gamma \xi}}{1 - e^{4\gamma l}} \right) d\xi$$

$(\xi < x_1 < +l)$

Using formulas (6), (12) - (14), we study the stresses in the connection for the following most interesting cases [10-12].

1. Switching on is free from external loads, the matrix is stretched by stress p_{11} in an unperturbed field.

In this case

$$\tau = \frac{P_{11}}{\Delta \delta \gamma^2 (1 + \nu^-)} \cdot \left(1 - \frac{ch \gamma x_1}{ch \gamma l} \right) \quad (24)$$

$(|x_1| < l)$

$$\tau = \frac{P_{11}}{\Delta 2\gamma (1 + \nu^-) \Delta} \cdot \frac{j h \gamma x_1}{ch \gamma l}$$

Diagrams of stresses $\tau(x_1)$ and $\tau(x_1)$ in this case are shown in Fig. 4.

The greatest value of the voltage in the inclusion $\tau = \tau_{max}$ is achieved in its middle at $x_1 = 0$.

$$\tau_{max} = \frac{P_{11}}{\Delta \delta \gamma^2 (1 + \nu^-)} \cdot \left(1 - \frac{1}{ch \gamma l} \right) \quad (25)$$

The tangential stress $\tau(x_1)$ at this point vanishes.

Using (7) we transform the divisor value

$$\Delta \delta \gamma^2 (1 + \nu^-) = \frac{E^-}{E^+} = \frac{1}{\lambda} \quad (26)$$

Therefore, we have $(\lambda > l)$:

$$\tau = \lambda P_{11} \cdot \left(1 - \frac{ch \gamma x_1}{ch \gamma l} \right); \quad \tau_{max} = \lambda P_{11} \cdot \left(1 - \frac{1}{ch \gamma l} \right) \quad (27)$$

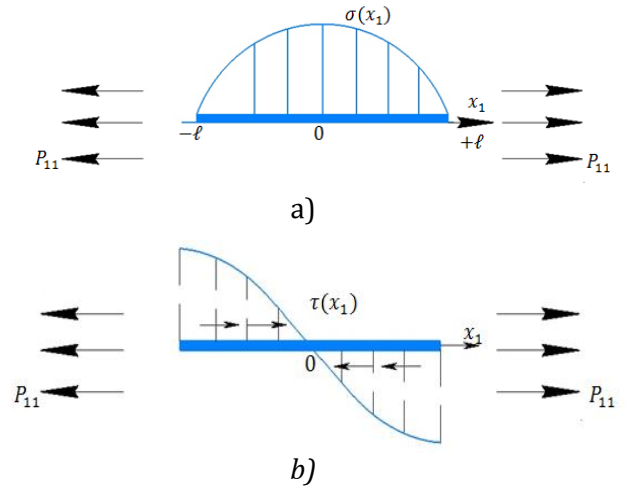


Fig. 4.

Let us dwell in more detail on two limiting cases:

very wide inclusion when $\lambda \gg l$

$$\tau_{max} = \lambda p_{11} \quad (28)$$

ideal-hard switching when $\lambda \ll l$

$$\tau_{max} = \frac{1}{2} \lambda p_{11} \gamma^2 l^2 = \frac{l^2 P_{11}}{2 \Delta \delta (1 + \nu^-)} \quad (29)$$

As you can see, the wider and tougher the thin inclusion, the greater the stresses are concentrated in it; in this case, in the disturbed zone (boundary layer), the matrix is completely

freed from normal stresses. This reinforcement effect is practically used in composite materials. Switching voltages can be thousands of times higher than matrix voltages [13-17].

However, the possible effect of the inclusion width is limited by the tensile strength of the inclusion, since an inclusion that is too wide breaks in half in the middle. Let us find the maximum width of the inclusion $l=l_{max}$, at which in its middle, at the point $x_1=0$, the limiting tensile strength of the inclusion is achieved $\tau=\tau_B$; using (29), we find

$$l_{max} = \frac{1}{\gamma} \operatorname{arch} \left(1 - \frac{\tau_B}{\lambda P_{11}} \right)^{-1} \quad (30)$$

$$0 < l < l_{max}$$

Thus, the greater the Young's modulus of the inclusion and its tensile strength, the greater the reinforcing effect.

2. Tensile forces p are applied to both ends of the inclusion, there is no external field in the matrix ($p_{11} = 0$).

In this case, we have

$$\tau = \frac{P_{11}}{\delta} \cdot \frac{ch\gamma x_1}{ch\gamma l} \quad (31)$$

$$\tau = \frac{-P\gamma}{2} \cdot \frac{jh\gamma x_1}{ch\gamma l} \quad |x_1| < l$$

The stress diagrams τ and τ in this case are shown in Fig. 5.

The greatest value of stress τ is achieved at the ends of the inclusion, therefore, the possible magnitude of the force p is limited by the strength of the inclusion at break $p \leq \delta \tau_B$.

3. Force p is applied only to the right end of the inclusion, there is no external field in the matrix ($p_{11}=0$).

In this case, we have

$$\tau = \frac{P}{\delta} \cdot \left(\frac{ch\gamma x_1}{ch\gamma l} + \frac{jh\gamma x_1}{jh\gamma l} \right) \quad (32)$$

$$\tau = \frac{-P\gamma}{\delta} \cdot \left(\frac{jh\gamma x_1}{ch\gamma l} + \frac{ch\gamma x_1}{jh\gamma l} \right) \quad (|x_1| < l)$$

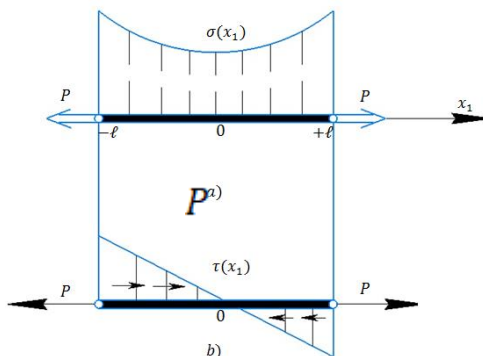


Fig. 5.

Diagrams of stresses τ and τ in the inclusion are shown in Fig. 6.

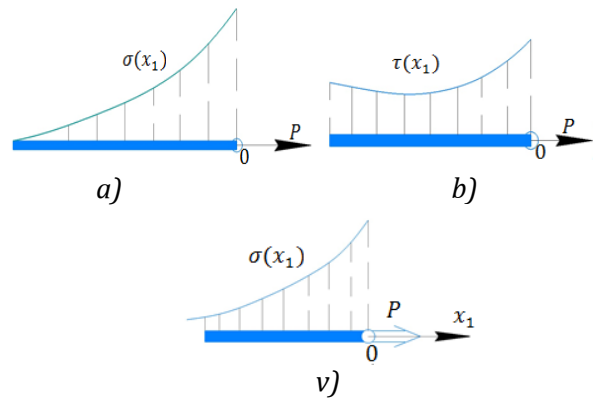


Fig. 6.

Consider one interesting limiting case of this problem, when the width of the inclusion tends to infinity ($\gamma l \rightarrow \infty$).

First, in formula (32), we transfer the origin of coordinates to the right end (Fig. 6), we obtain

$$\tau = \frac{P}{2\delta} \cdot \left(\frac{ch\gamma(l+x_1)}{ch\gamma l} + \frac{jh\gamma(l+x_1)}{jh\gamma l} \right) \quad (x_1 < 0) \quad (33)$$

For a very wide inclusion, when ($\gamma l \rightarrow \infty$), from this we find

$$\tau = \frac{P}{\delta} e^{\gamma x_1}; \quad \tau = -\frac{1}{2} p \gamma e^{\gamma x_1} \quad (34)$$

$$(\gamma l \gg 1, \quad x_1 < 0)$$

As can be seen, the perturbation from the external force p decays at a distance of the order of $(3/\gamma)$ from the end of the inclusion [18-22]

4. Conclusions

We consider the singular problem of stretching an elastic plane along a rigid inclusion of high thermal conductivity and finite length, coupled to the rest of the body by the matrix at all points of the lateral surface, which is conjugate to the matrix. A general technique for solving such problems is built, qualitative and quantitative estimates are given to the stress states of the inclusion and matrix in the following loaded methods:

- switching on is free from external loads;
- external loads are applied to both ends of the connection;
- an external force is applied only to one of the ends of the inclusion;
- external force is applied to an arbitrary switching point.

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